

The Mathematics of Gerrymandering



Vetri Velan

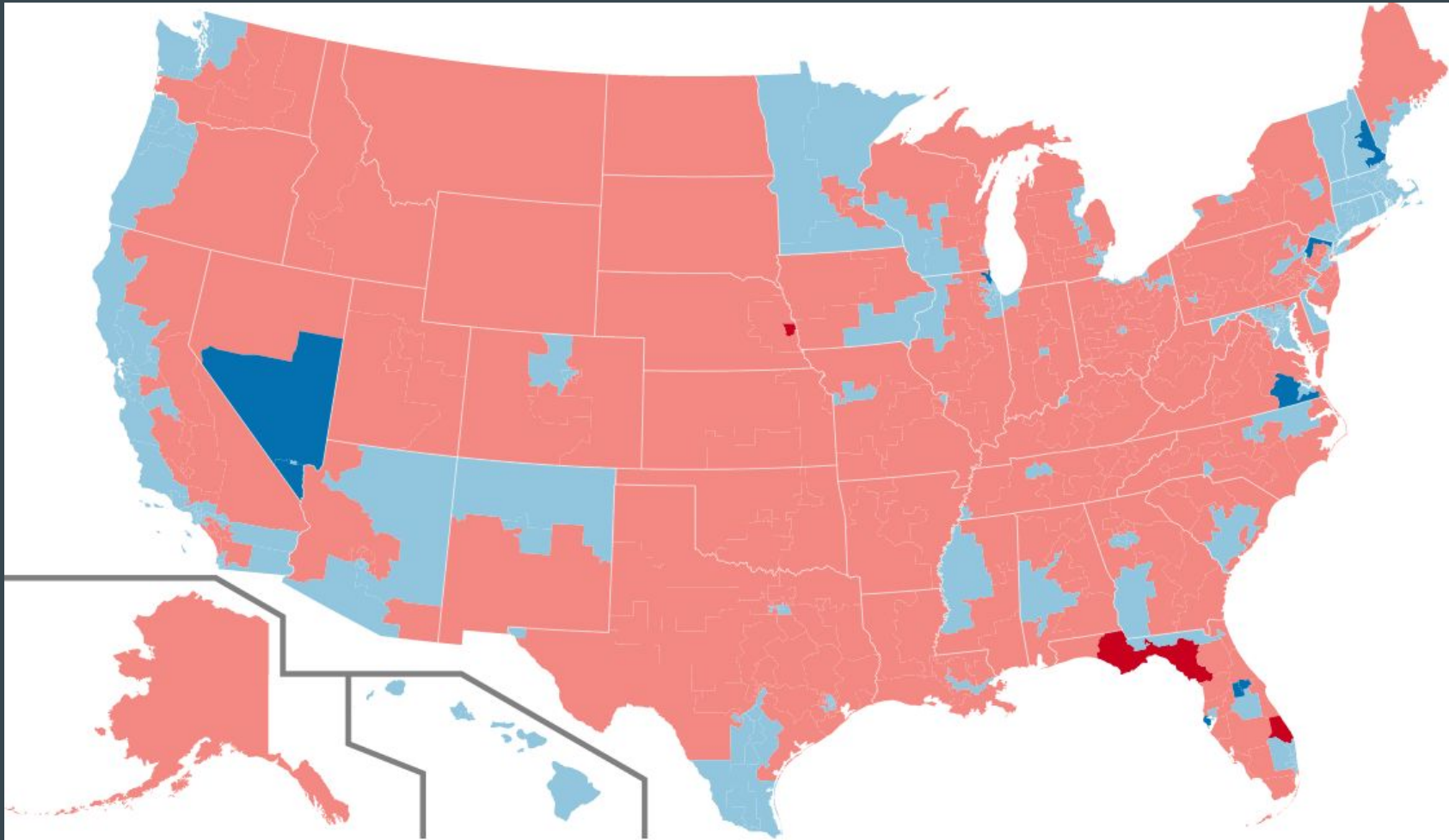
University of California, Berkeley

Physics Department- Grad Student Seminar

March 16, 2018

Article 1, Section 2. Paragraph 1

The House of Representatives shall be composed of Members chosen every second Year by the People of the several States, and the Electors in each State shall have the Qualifications requisite for Electors of the most numerous Branch of the State Legislature.



Wikipedia: United States House of Representatives elections, 2016

What is Gerrymandering?



Definition of gerrymander

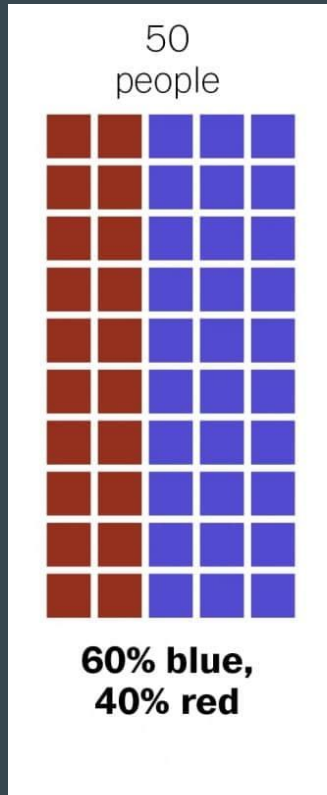
gerrymandered; gerrymandering \-d(ə-)rɪŋ\

transitive verb

1 : to divide (a territorial unit) into election districts to give one political party an electoral majority in a large number of districts while concentrating the voting strength of the opposition in as few districts as possible

2 : to divide (an area) into political units to give special advantages to one group
gerrymander a school district

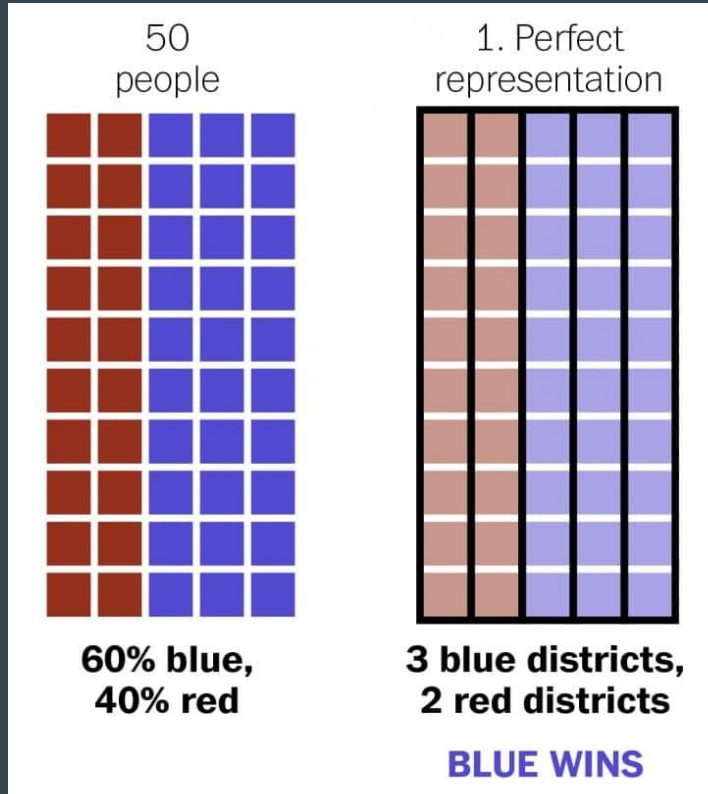
What is Gerrymandering?



Washington Post. "This is the best explanation of gerrymandering you will ever see" C. Ingraham, March 1, 2015.

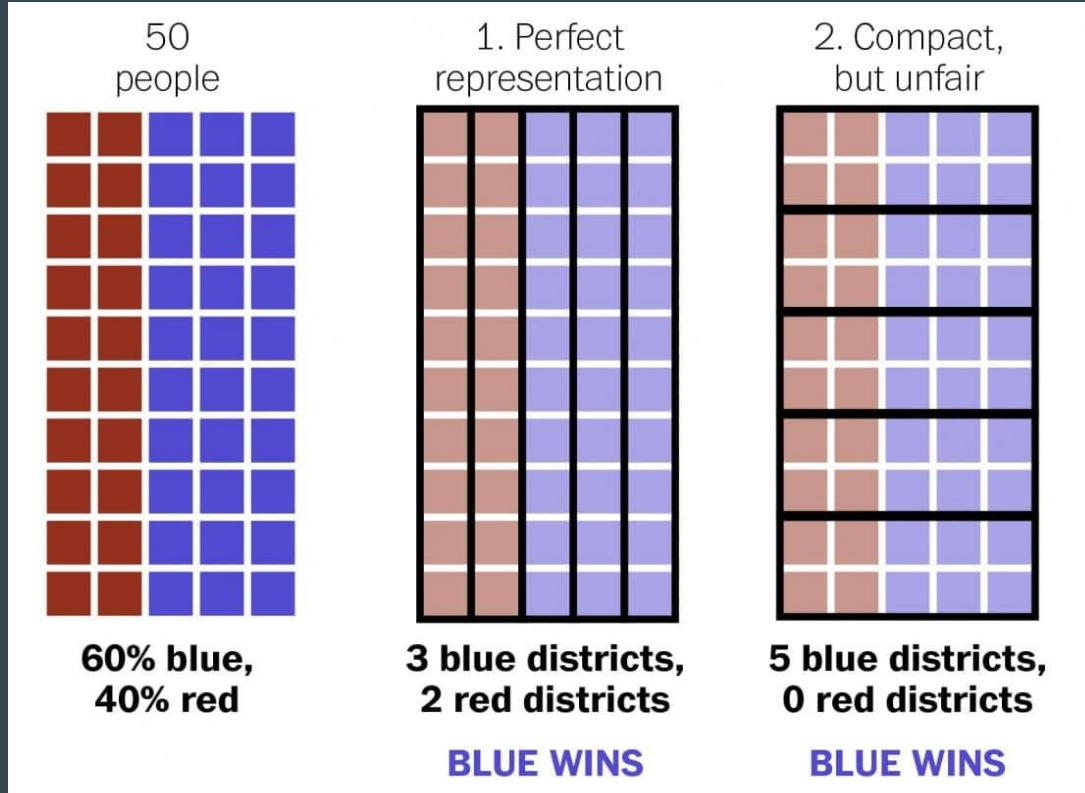
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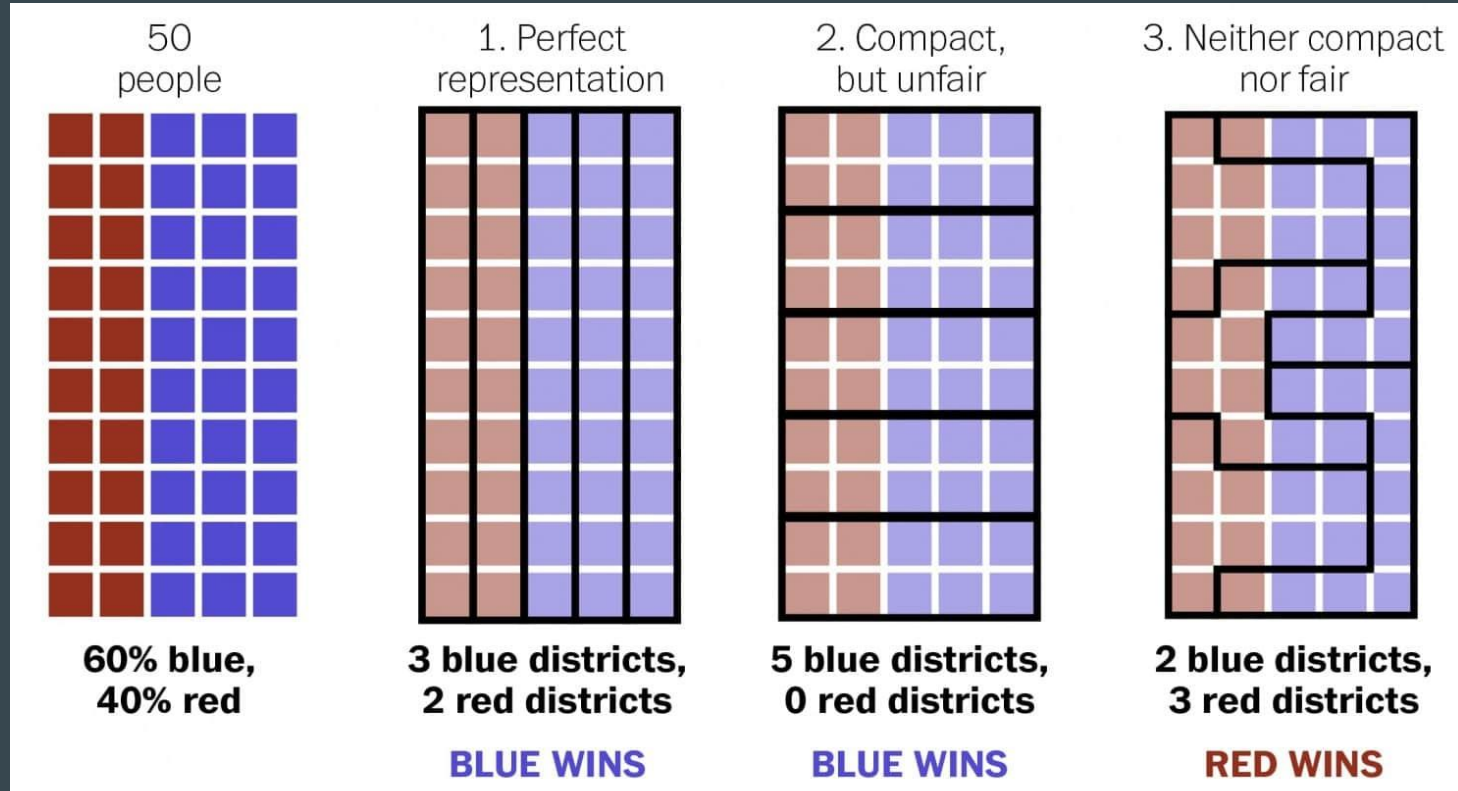
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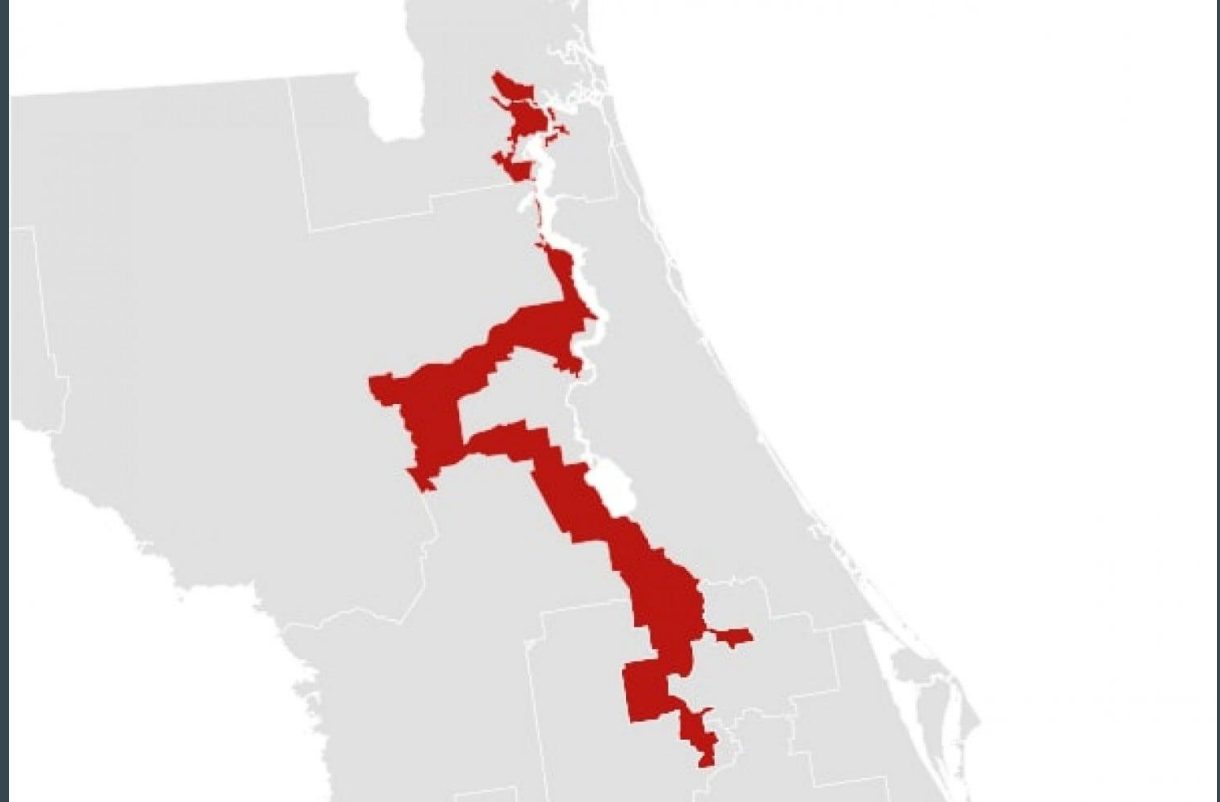


How can we measure gerrymandering?

1. Common sense: “If it looks like a duck...”

How can we measure gerrymandering?

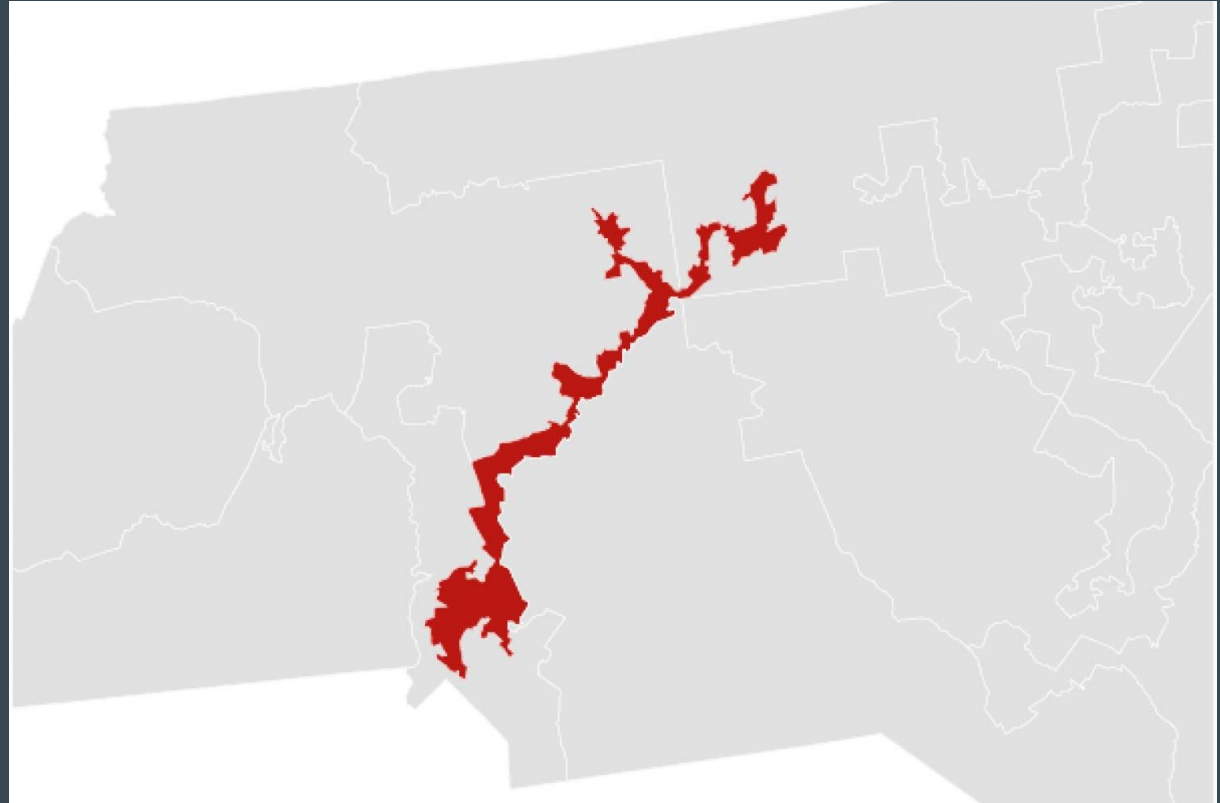
Florida
5th district



Washington Post. "America's most gerrymandered congressional districts".
C. Ingraham. May 15, 2014.

How can we measure gerrymandering?

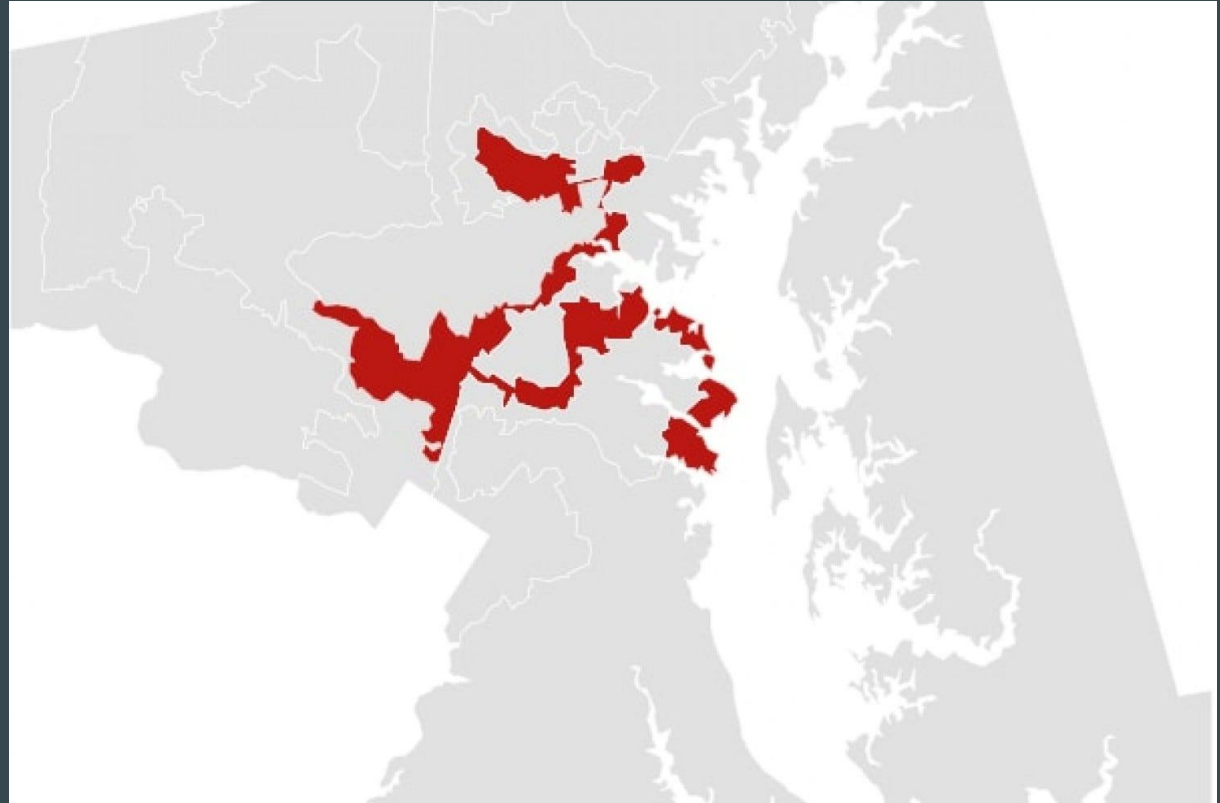
North Carolina
12th district



Washington Post. "America's most gerrymandered congressional districts".
C. Ingraham. May 15, 2014.

How can we measure gerrymandering?

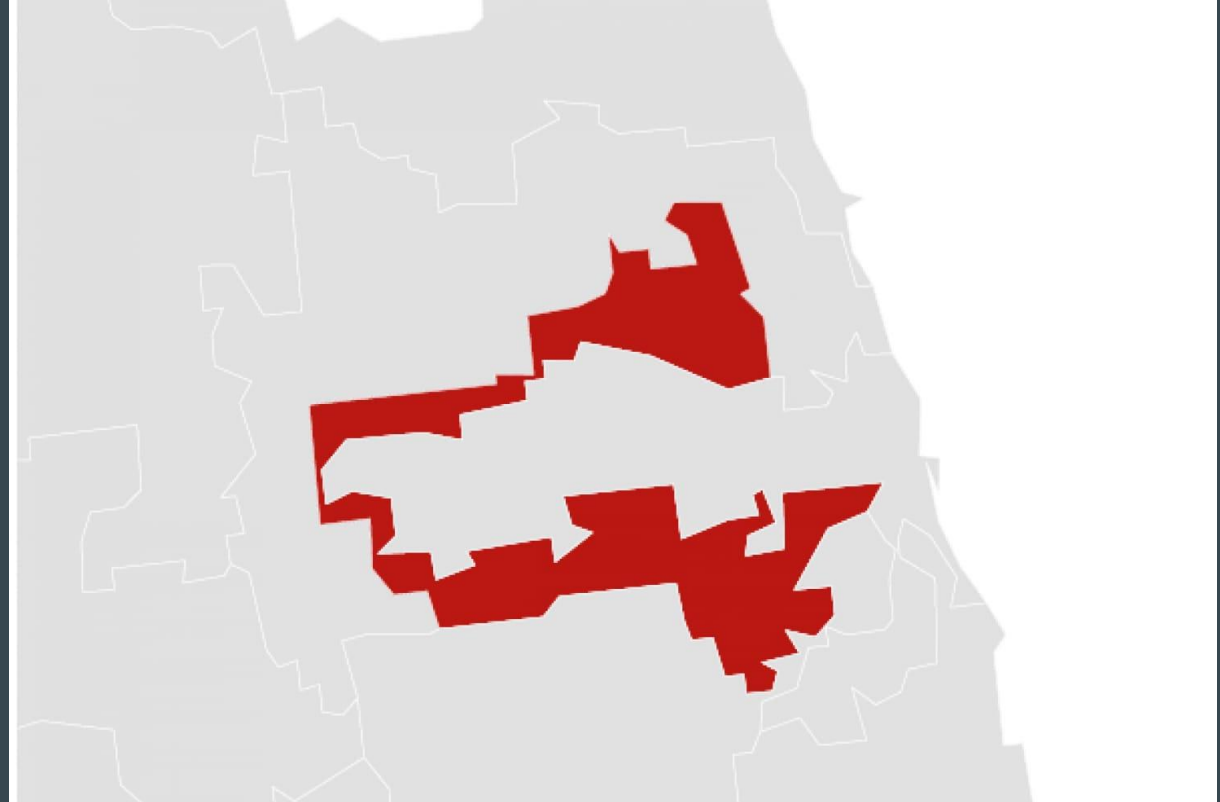
Maryland
3rd district
“The Praying
Mantis”



Washington Post. “America’s most gerrymandered congressional districts”.
C. Ingraham. May 15, 2014.

How can we measure gerrymandering?

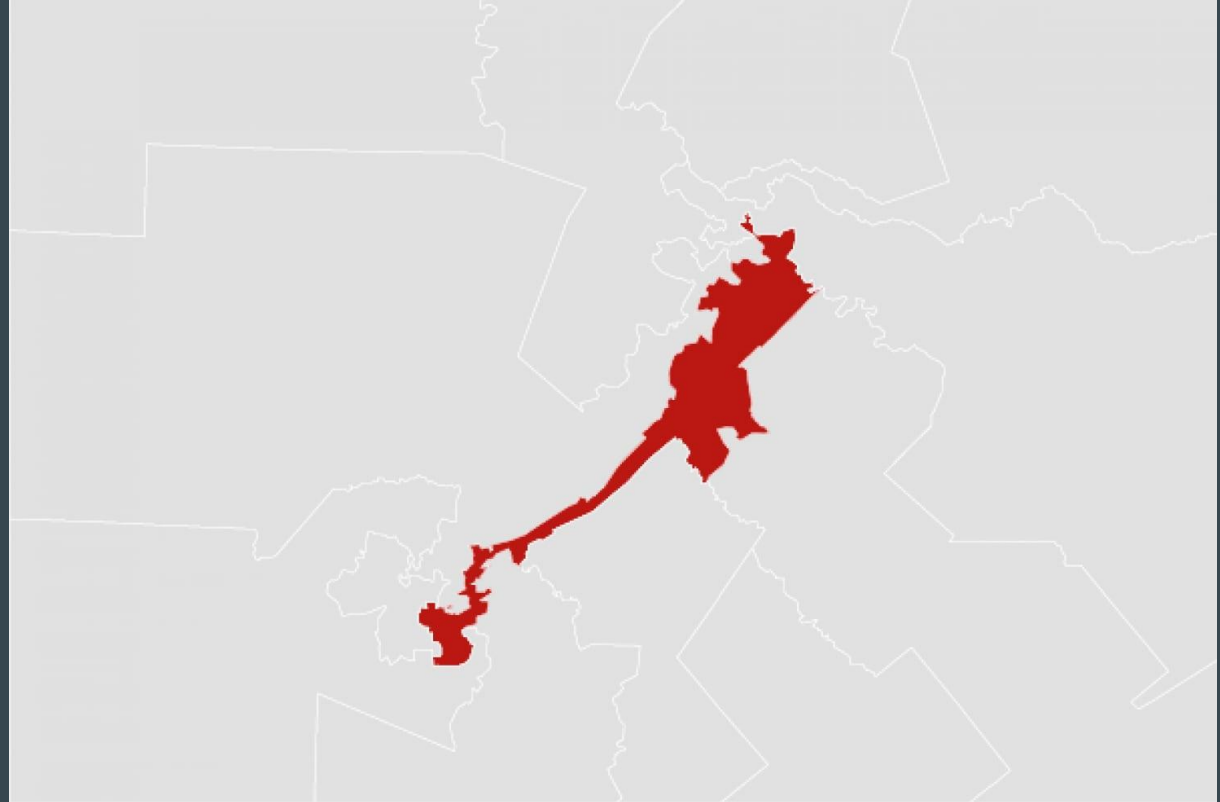
Illinois
4th district
“Earmuffs”



Washington Post. “America’s most gerrymandered congressional districts”.
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How can we measure gerrymandering?

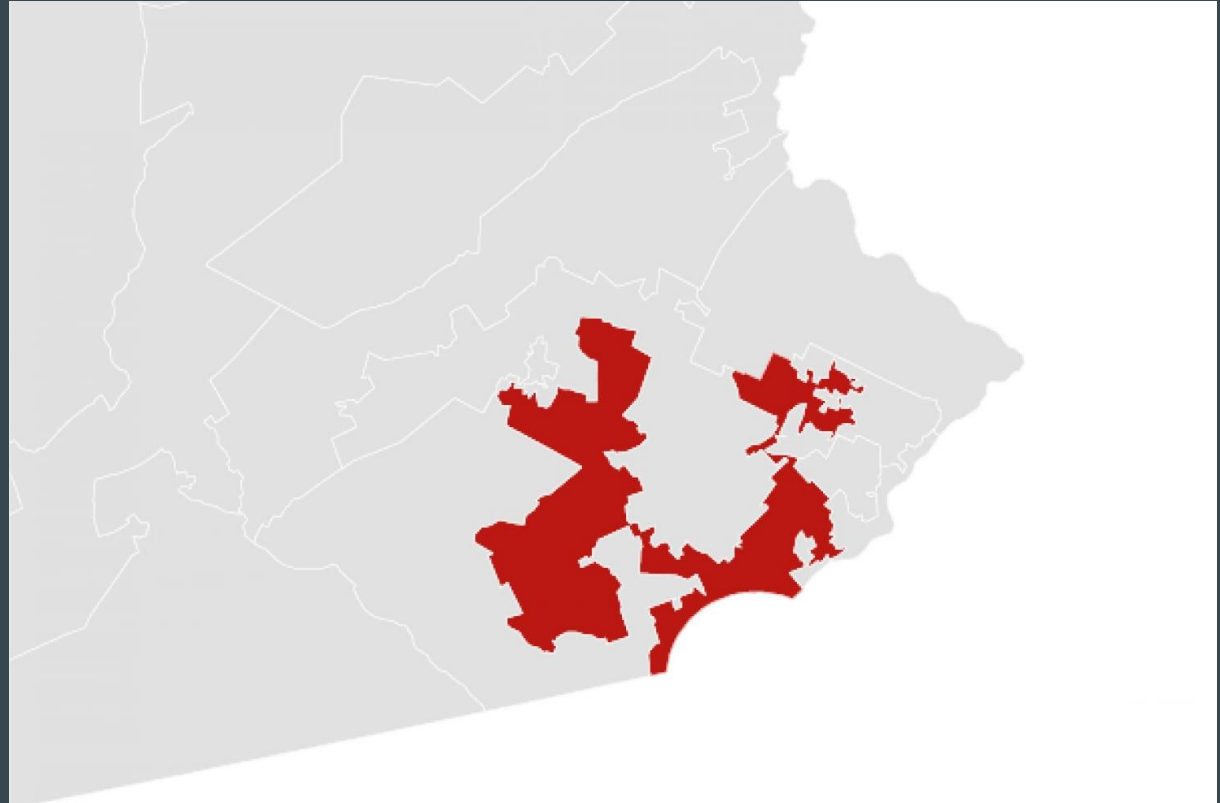
Texas
35th district
“The Upside
Down Elephant”



Washington Post. “America’s most gerrymandered congressional districts”.
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How can we measure gerrymandering?

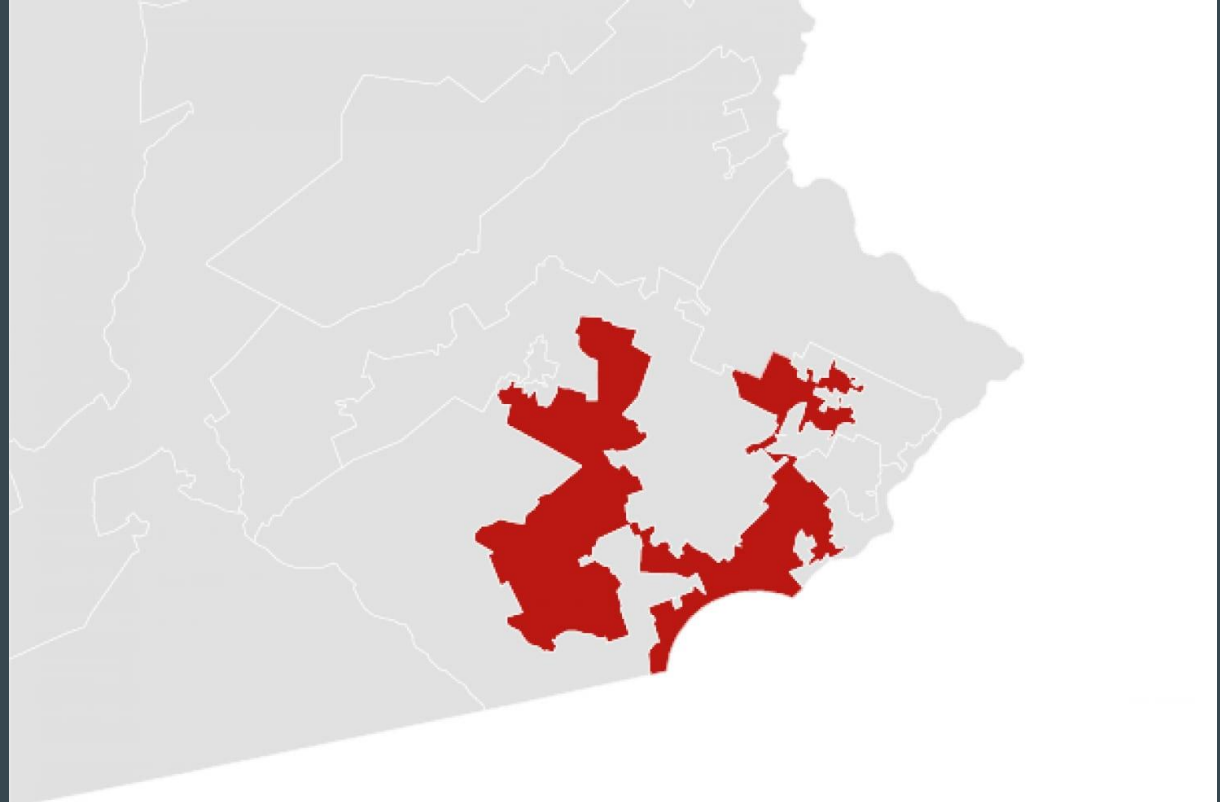
Pennsylvania
7th district



Washington Post. "America's most gerrymandered congressional districts".
C. Ingraham. May 15, 2014.

How can we measure gerrymandering?

Pennsylvania
7th district
“Goofy Kicking
Donald Duck”



Washington Post. “America’s most gerrymandered congressional districts”.
C. Ingraham. May 15, 2014.

How can we measure gerrymandering...mathematically?

- Requires a mathematical definition of gerrymandering
- Let's try to build one!
 - Prudent to think about the following question: If you were on a redistricting commission and wanted to gerrymander a state for your party, how would you do it?

How can we measure gerrymandering...mathematically?

- Requires a mathematical definition of gerrymandering
- Let's try to build one!
 - Prudent to think about the following question: If you were on a redistricting commission and wanted to gerrymander a state for your party, how would you do it?
- Generally, you want to *waste* the other party's votes
 - "Packing" and "Cracking"

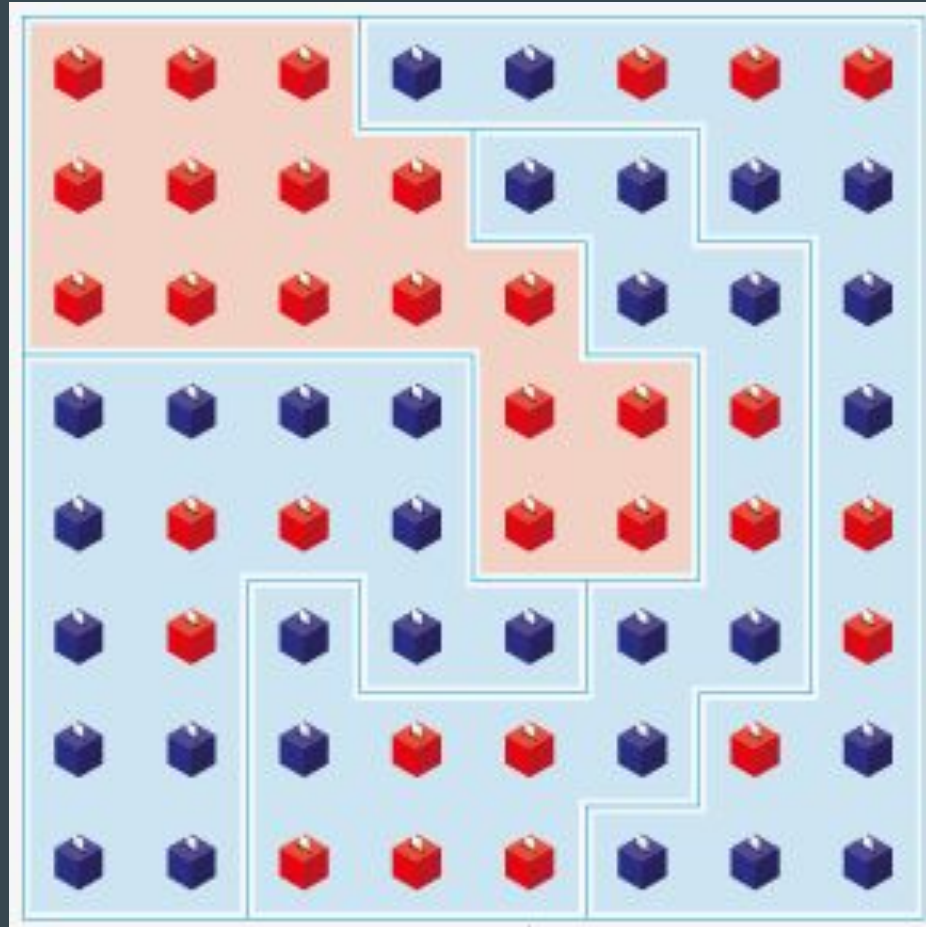
Packing

64 voters

32 R, 32 B

4 districts with
16 people each

R voters
“packed” into a
single district



District 1:
16 R, 0 B
R likely to win

District 2:
6 R, 10 B
B likely to win

District 3:
7 R, 9 B
B likely to win

District 4:
3 R, 13 B
B likely to win

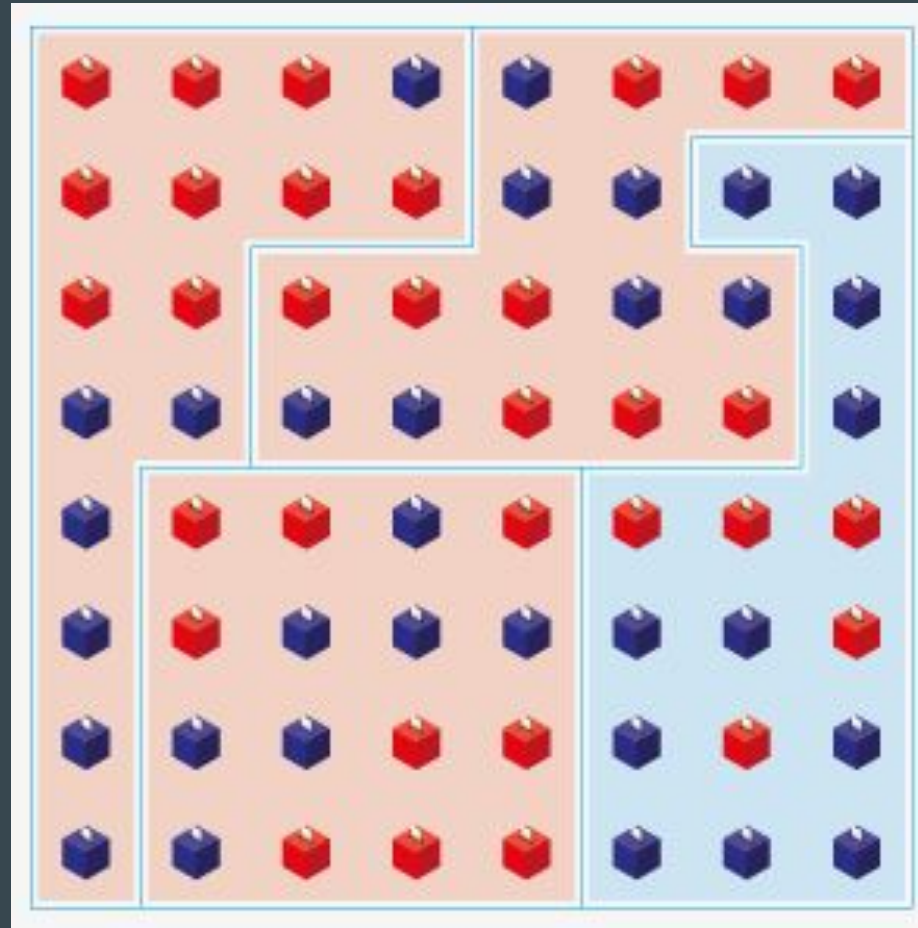
Cracking

64 voters

32 R, 32 B

4 districts with
16 people each

B voters
“cracked”
among many
districts



District 1:
9 R, 7 B
R likely to win

District 2:
9 R, 7 B
R likely to win

District 3:
9 R, 7 B
R likely to win

District 4:
5 R, 8 B
B likely to win

Packing and Cracking

- When districts are packed and/or cracked for one party, the other party is wasting its votes!
- Example:
 - Town voting for mayor, 100 people vote
 - 54 vote for Smith, 35 vote for Jones, 11 vote for Harris
 - All of the Jones and Harris voters have wasted their votes, because their candidate lost
 - Smith only needed 36 votes to win, so $54 - 36 = 18$ Smith voters wasted their votes
 - A majority of people (64%) have wasted their votes
- Can we use this principle to find a definition of gerrymandering?

How can we measure gerrymandering?

1. Common sense: “If it looks like a duck...”
2. Efficiency Gap

Efficiency Gap

Democrats:

159k - 186k =

26k wasted votes

Republicans:

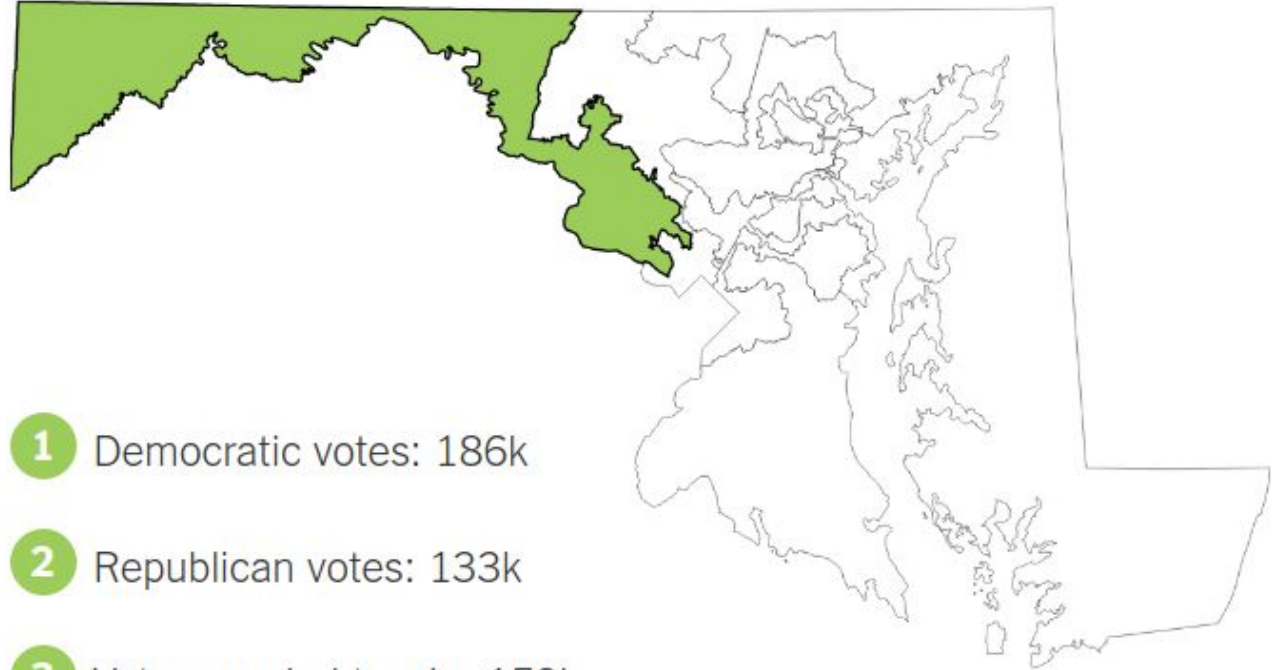
133k wasted votes

Net:

107k wasted R votes

14% of D votes and

100% of R votes



New York Times. "How the New Math of Gerrymandering Works." N. Cohn and Q. Bui. Oct 3, 2017.

Efficiency Gap

Maryland-wide:
1,636k Dem votes,
962k Rep votes

With proportional
representation:
5 D seats, 3 R seats

Real life:
7 D seats, 1 R seat

District	Dem. votes	G.O.P. votes	votes to win	Wasted Votes		
				for Dem.	for G.O.P.	Net
01	104k	243k	173k	104k	69k	34k Dem.
02	192k	103k	147k	45k	103k	58k GOP
03	215k	115k	165k	50k	115k	65k GOP
04	238k	69k	153k	84k	69k	16k Dem.
05	243k	106k	174k	69k	106k	37k GOP
06	186k	133k	159k	26k	133k	107k GOP
07	239k	70k	154k	85k	70k	15k Dem.
08	221k	125k	173k	48k	125k	77k GOP
TOTAL	1,636k	962k	1,299k	510k	789k	279k GOP

Efficiency Gap

Maryland-wide:
1,636k Dem votes,
962k Rep votes

With proportional
representation and
binomial statistics:
 5.0 ± 1.4 Dem seats,
 3.0 ± 1.4 Rep seats

Real life:
7 D seats, 1 R seat

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Efficiency Gap

Wasted votes:
510k D and 789k R

279k more R votes
than D votes wasted

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Efficiency Gap

Wasted votes:

510k D and 789k R

279k more R votes
than D votes wasted

Efficiency gap:

Net votes wasted /
Total votes cast

In MD:

279k / (1636k + 962k)
= 10.7% in favor of D

District	Dem. votes	G.O.P. votes	votes to win	Wasted Votes		
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Efficiency Gap

What efficiency gap counts as gerrymandering?

No “one-size-fits-all” answer

E. McGhee and N. Stephanopoulos, creators of efficiency gap, propose a 7% standard to serve as evidence of gerrymandering

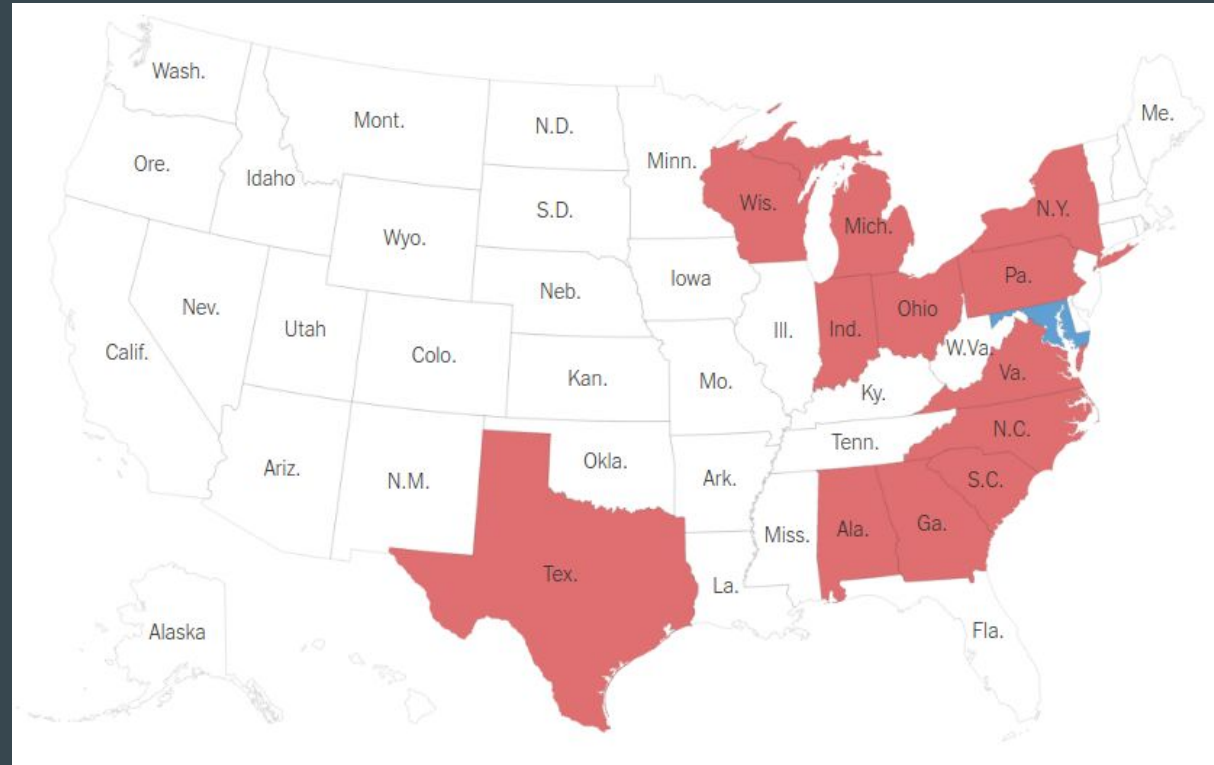
Efficiency Gap

States that have at least 5 districts and exceed 7% efficiency gap based on 2016 Congressional race

“Smell test”:

WI, PA, NC all currently facing litigation

VA, MD often regarded as gerrymandered



New York Times. “How the New Math of Gerrymandering Works.”
N. Cohn and Q. Bui. Oct 3, 2017.

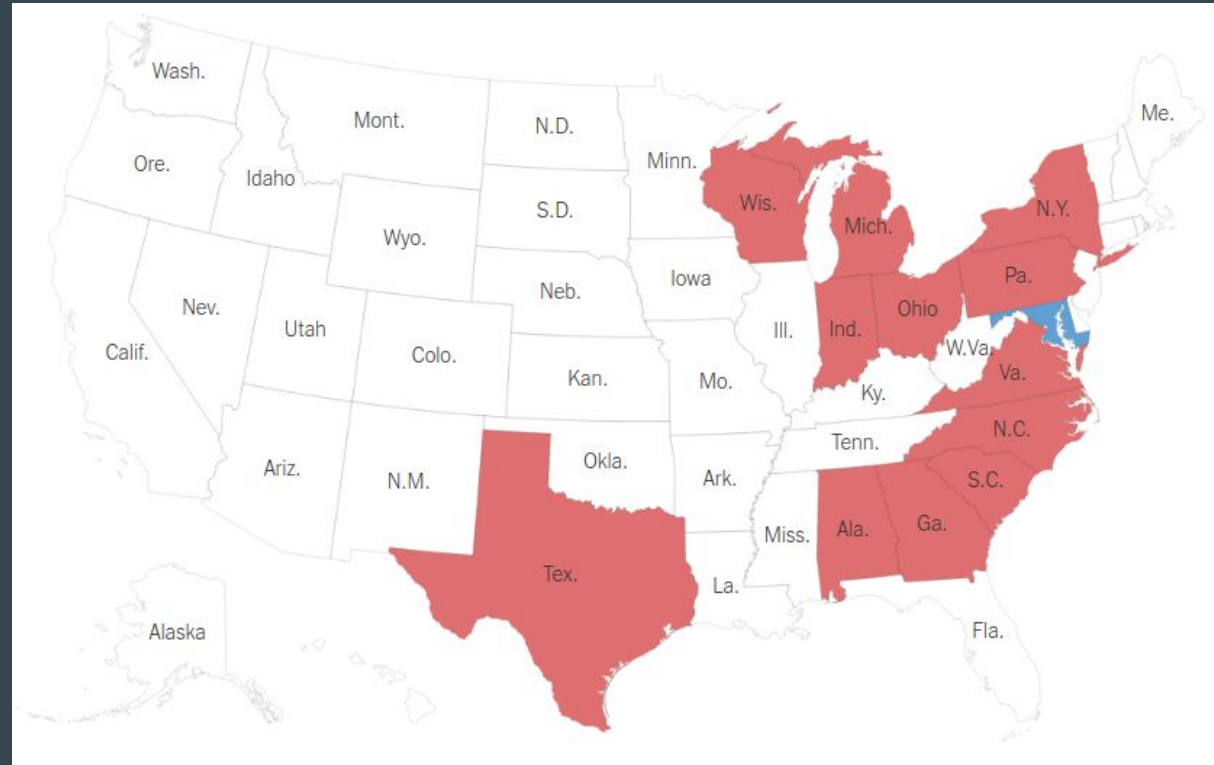
Efficiency Gap

Problems:

Illinois regarded as Dem gerrymander, but eff. gap slightly favors Republicans

Indiana regarded as fair, but classified as gerrymandered

New York has efficiency gap of >7% towards Republicans (???)



New York Times. "How the New Math of Gerrymandering Works." N. Cohn and Q. Bui. Oct 3, 2017.

Efficiency Gap

Another problem: it doesn't work for small states

Rhode Island Congressional Delegation

(elected 2016)

The **Democratic Party** had a **19.8% efficiency gap advantage*** worth **0 extra seats**.

65% Democratic vote

35% Republican vote



2 Democratic seats



DATA ANALYTICS
azavea

* The "efficiency gap" measures how effectively a party's votes are distributed among districts and reveals partisan bias.

Azavea blog. "The Most Gerrymandered States Ranked by Efficiency Gap and Seat Advantage". D. McGlone and E. Needham. July 19, 2017.

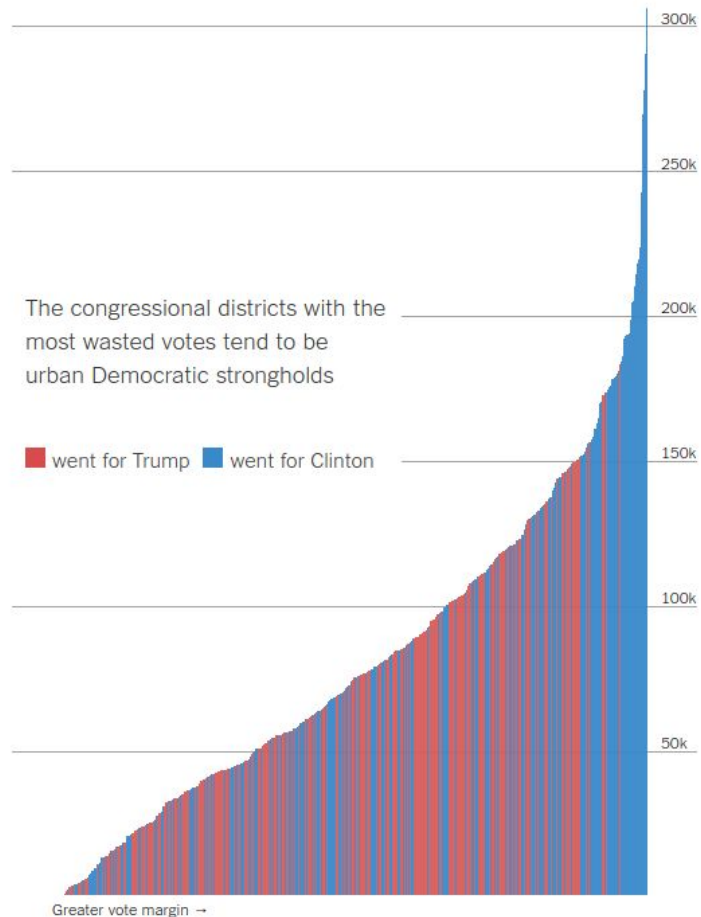
Efficiency Gap

Political geography

Self-sorting into Democratic and Republican “bubbles” --especially for Democrats

Number of wasted votes in victory by congressional district

2016 presidential election



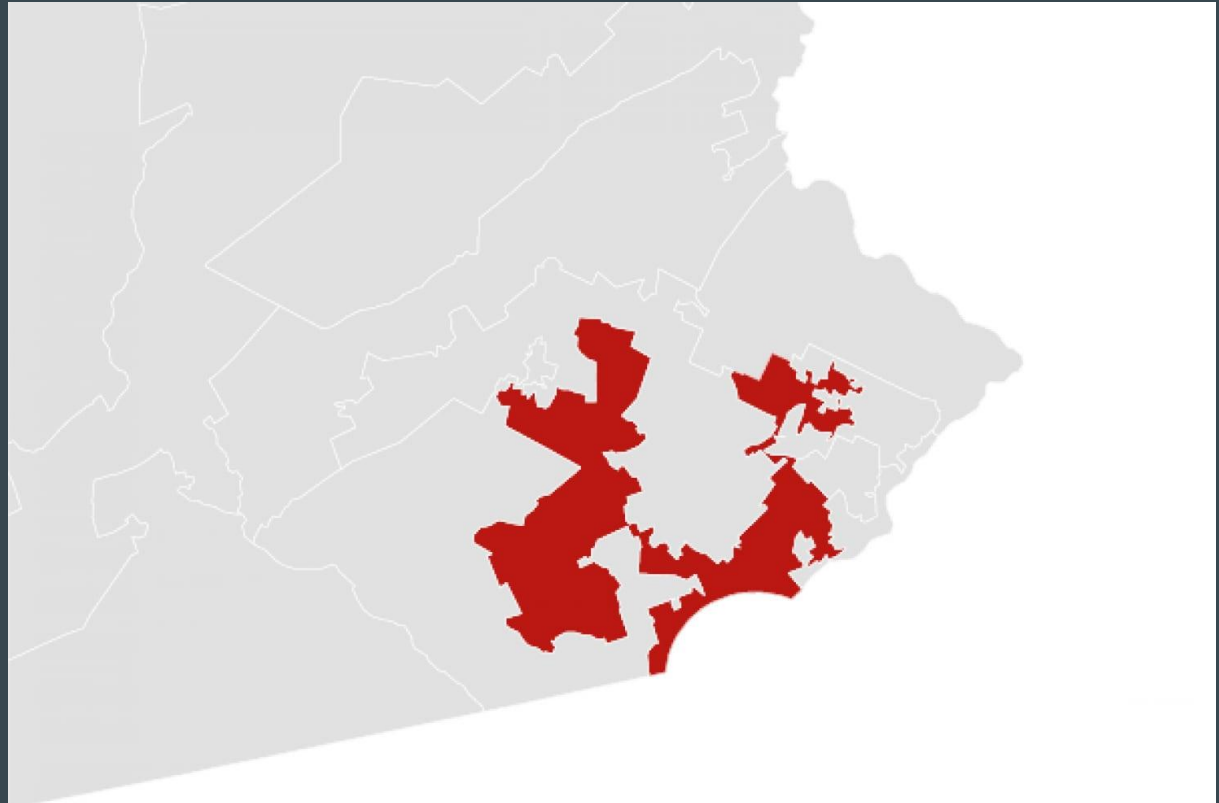
How can we measure gerrymandering?

1. Common sense: “If it looks like a duck...”
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3. District Compactness

District Compactness

Gerrymandered districts often have strange shapes

Measure how compact a district is using some geometric metric or algorithm



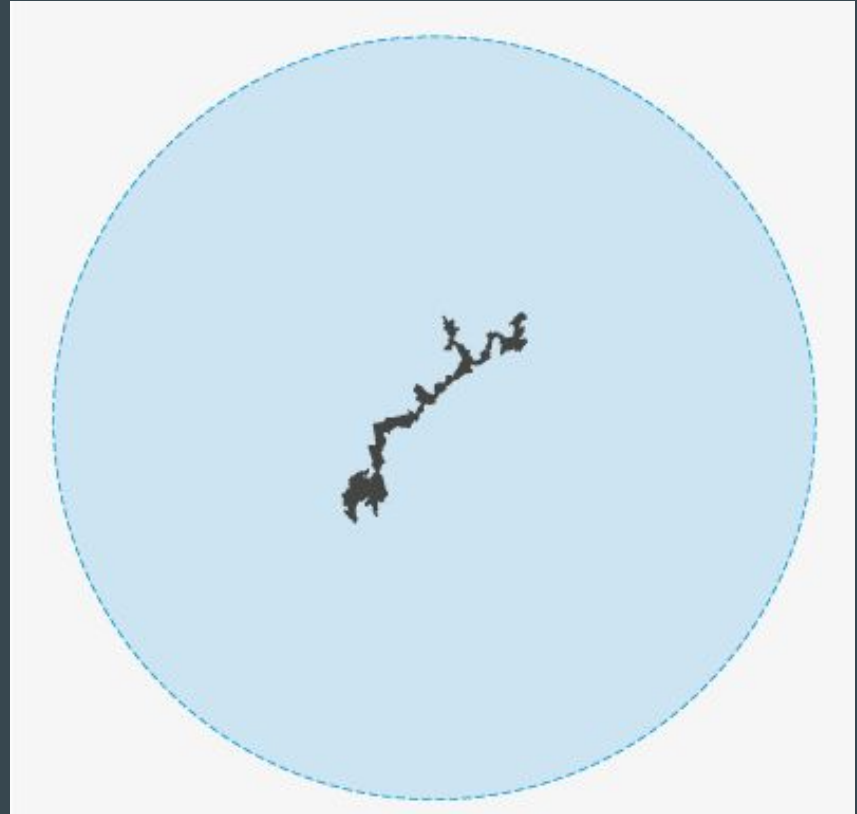
Washington Post. "America's most gerrymandered congressional districts".
C. Ingraham. May 15, 2014.

District Compactness

Polsby-Popper Method:

Ratio of district perimeter squared to its area

Circle has lowest perimeter to area ratio



District Compactness



Nature News. "The mathematicians who want to save democracy." C. Arnold. June 7, 2017.

Reock

Ratio of the district's area to that of the smallest possible circle that can enclose it

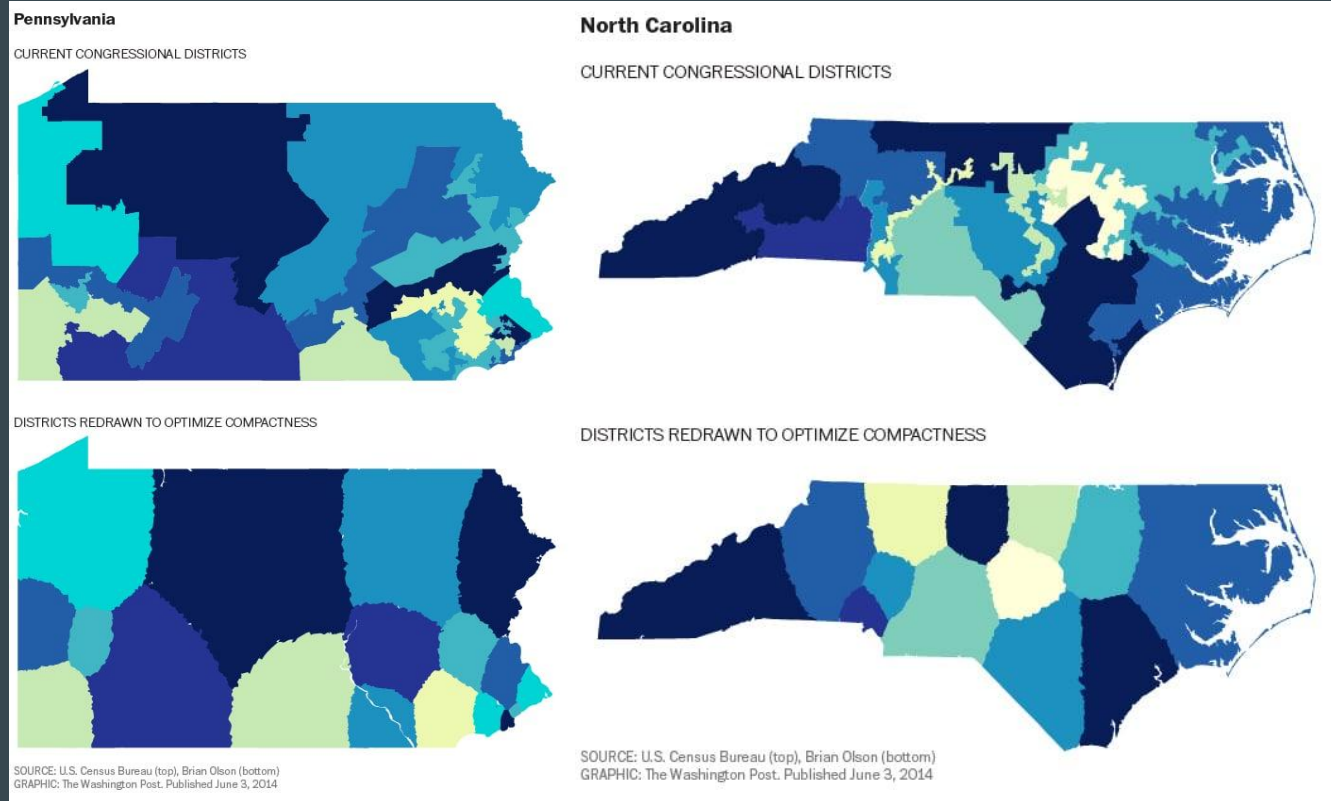
Area/Convex Full

Ratio of the district's area to that of the smallest simple polygon that can surround it

District Compactness

B. Olson's algorithm minimizes the distance between an average constituent and their district's geographic center, based on US Census 2010 Data

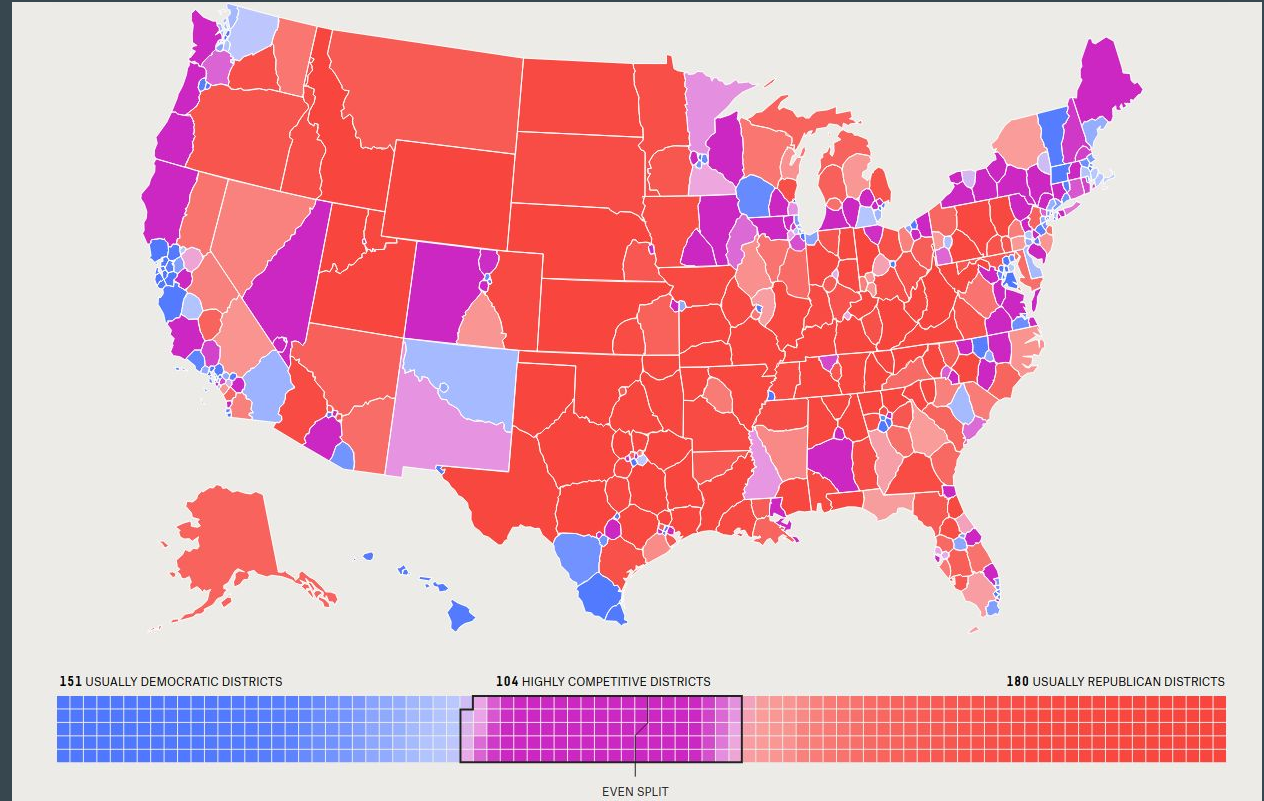
Washington Post. "This computer programmer solved gerrymandering in his spare time." C. Ingraham. June 3, 2014.



District Compactness

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FiveThirtyEight. "The Atlas Of Redistricting." A. Bycoffe, E. Koeze, D. Wasserman, and J. Wolfe. Jan 25, 2018.



District Compactness

Issues:

Geography is not compact: rivers, mountains, etc.

Communities are not compact: cities, suburbs, neighborhoods

How can we measure gerrymandering?

1. Common sense: “If it looks like a duck...”
2. Efficiency Gap
3. District Compactness
4. Simulations

Simulations

Simulate districting many times (Monte Carlo method), given some parameters

I.e. maximize compactness, competitive elections, communities of interest

Calculate a measure of gerrymandering for each simulation

Compare to actual data; is it consistent or an outlier?

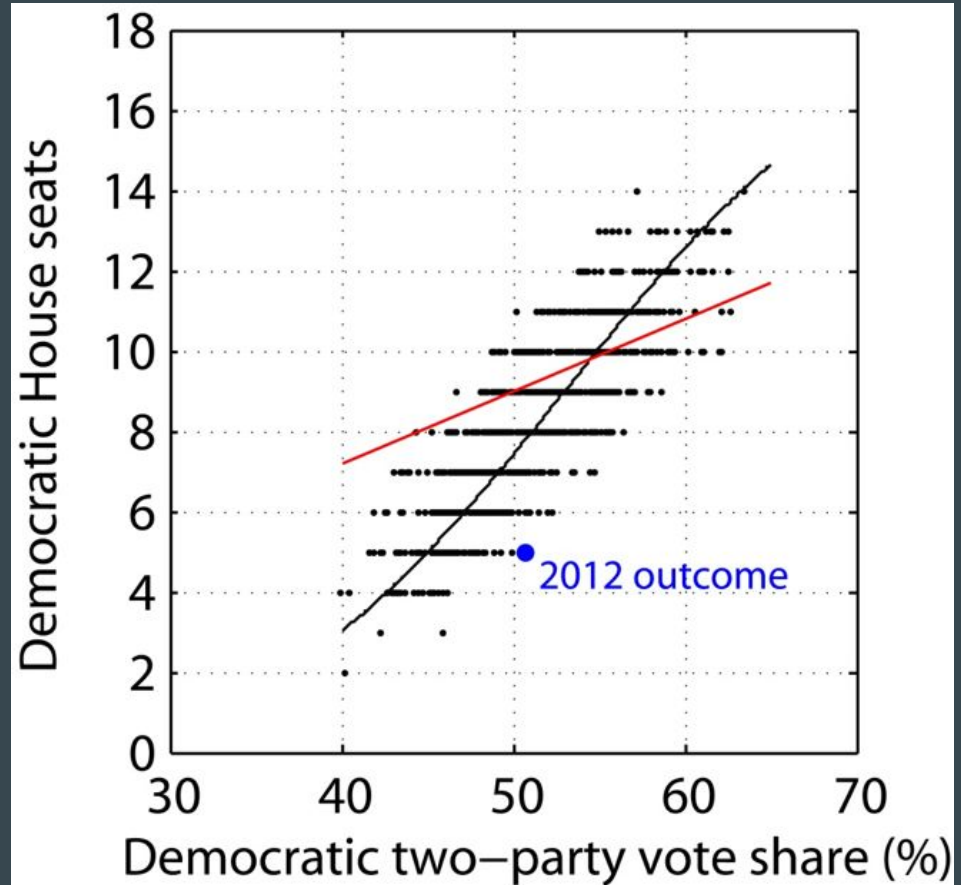
Essentially hypothesis testing

Simulations

The Princeton Gerrymandering Project

Method:

- PA has 18 districts
- Create pseudo-state: pick 18 random districts out of all 435
- Compare partisan makeup of pseudo-state to PA; keep if within 0.2%
- Record number of D/R seats within pseudo-state

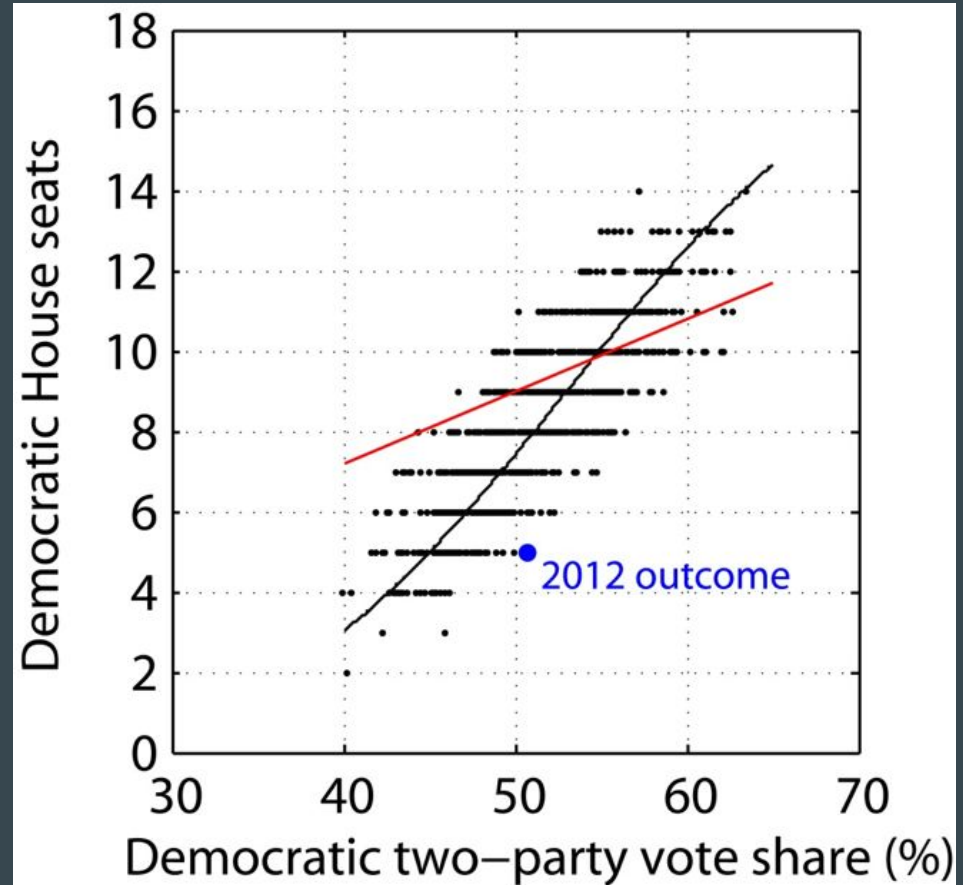


Simulations

The Princeton Gerrymandering Project

Results:

- In 2012, Dems won 51% of House vote in PA, and won 5/18 seats.
- This was fewer seats than any simulation in which Dems won 51% of the vote.



Simulations

W. Pegden at Carnegie Mellon

Method:

- Take the current map, perturb it slightly while keeping contiguity and population
- Compute difference between median and mean partisan vote across districts

Results:

- Out of $1e12$ simulations, only $1e4$ had a higher median vs. mean difference
- Suggests that districts were optimized to favor one party

Simulations

Issues:

Not standardized

Depends on interpretations,
which can be biased

Simulations

Issues:

Not standardized

Depends on interpretations,
which can be biased

Difficult to explain to lawyers,
judges, and politicians

```
1           CHIEF JUSTICE ROBERTS:  No, but you're
2 going to take this -- the whole point is you're
3 taking these issues away from democracy and
4 you're throwing them into the courts pursuant
5 to, and it may be simply my educational
6 background, but I can only describe as
7 sociological gobbledygook.
```

Conclusions

- No universal definition of gerrymandering, or remedy for it
- Many strategies that, taken together, can indicate intentional partisan districting
- Social science has a lot of overlap with physics! We build models, test them, and interpret them to the best of our understanding. When we get more information, we adapt our theories.
- Things I didn't include:
 - Is gerrymandering bad?
 - Racial gerrymandering
 - Legal challenges to gerrymandering

